THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2022**

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's Mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) The time taken by John to deliver milk to the High Street is normally distributed with mean 12 minutes and standard deviation 2 minutes. If he delivers milk every day, estimate the number of days during the year when he takes longer than 17 minutes. (1 year = 365 days)

   (b) Suppose that a group of people in a village attending hospital has been categorized according to the incidence of two diseases

Sex	Malaria	Typhoid
Male	16	12
Female	12	10

   Find the probability that the person chosen is a female given that the person is suffering from malaria.

   (c) In how many ways can a hand of 4 cards be chosen from an ordinary pack of 52 playing cards?

2. (a) Write the converse and inverse of the statement "If you score an A grade in a logic test, then I will buy you a new car" in words and symbolic form.

   (b) Using a truth table, examine whether $[(\sim p)\rightarrow(\sim q)]\wedge(p\rightarrow q)$ is equivalent to $(q\rightarrow p)\wedge(p\rightarrow q)$.

   (c) Use laws of algebra of propositions to simplify $[p\wedge(p\vee q)]\vee[q\wedge(\sim(p\wedge q))]$.

3. (a) Find the work done by force $\underline{F}=\underline{i}+2\underline{j}+\underline{k}$ moving an object at a distance of 7 m in the direction of the vector $\underline{r}=3\underline{i}+2\underline{j}+4\underline{k}$.

   (b) If P and Q are points $P(3,-4,6)$ and $Q(1,-3,8)$ respectively, find a unit vector parallel to the displacement vector $\overrightarrow{PQ}$.

   (c) The position vectors of points A and B are $\underline{a}$ and $\underline{b}$ respectively. If point C divides $\overrightarrow{AB}$ internally in the ratio of $2:1$, D divides $\overrightarrow{AB}$ externally in the ratio of $1:4$ and E divides $\overrightarrow{CD}$ internally in the ratio of $2:1$, find the position vectors of C, D and E in terms of $\underline{a}$ and $\underline{b}$.

4. (a) Express $\sqrt{1+i}$ in polar form.

   (b) Using the results in part (a), show that $\tan\frac{\pi}{8}=\sqrt{2}-1$.

   (c) If $z_1=r_1(\cos\theta_1+i\sin\theta_1)$ and $z_2=r_2(\cos\theta_2+i\sin\theta_2)$; Prove that $Arg\left(\frac{z_1}{z_2}\right)=Arg(z_1)-Arg(z_2)$.

   (d) The complex numbers $z_1=\frac{c}{1+i}$ and $z_2=\frac{d}{1+2i}$ where $c,d\in\mathbb{R}$ are such that $z_1+z_2=1$, find the values of c and d.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) For all values of α show that $\frac{\sin 3\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\cos\alpha} = 2$.

(b) Prove that $\frac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \tan 2x$.

(c) Solve for β in the trigonometric equation $\tan^{-1}\left(\frac{\beta-1}{\beta-2}\right) + \tan^{-1}\left(\frac{\beta+1}{\beta+2}\right) = \frac{\pi}{4}$.

(d) Rewrite $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta-\alpha)$, hence solve the equation $4\cos\theta + 3\sin\theta = \frac{5\sqrt{2}}{2}$ in the interval $\frac{\pi}{2} \le \theta \le 2\pi$.

6. (a) If the coefficients of x and x^2 in the expansion of $\frac{1+px+qx^2}{(1-x)^2}$ are zero, find the numerical values of p and q

(b) Use the principle of mathematical induction to prove that for every positive integer, $3^{2n-2} + 2^{6n}$ is divisible by 5.

(c) Given that $P(x) = 2x^3 + 7x^2 - 5$, use the synthetic method to find the quotient and remainder when $P(x)$ is divided by $x+3$.

(d) Find the determinant and inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, hence solve the simultaneous equations $\begin{cases} 2x + y = 4 \\ x + 5y + 2z = 7 \\ x - y + z = 1 \end{cases}$.

7. (a) (i) Determine the most general function $M(x,y)$ such that the differential equation $M(x,y)dx + (2x^2y^3 + x^4y)dy = 0$ is exact.

(ii) By separating the variables, solve the differential equation $(xy + x)dx - (x^2y^2 + x^2 + y^2 + 1)dy = 0$

(b) Find the general solution of the differential equation $\cos x \frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} = 0$.

(c) A liquid of $72\,^0C$ placed in a room at $25\,^0C$ has a temperature of $65\,^0C$ after 5 minutes. Find its temperature after further 10 minute.

(d) Formulate a differential equation of a circle which passes through the origin and whose centre lies on the y-axis.

8. (a) Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$.

(b) (i) Determine the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a\cos\theta, b\sin\theta)$.

(ii) If the normal in part (b) (i) meets the x-axis at A and the y-axis at B, find the area of the triangle AOB where O is the origin.

(c) Show that the equation of the tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $x - ty + at^2 = 0$.

(d) (i) Change the Cartesian equation $(x^2 + y^2)^3 - 2xy(x^2 - y^2)$ into a polar equation.

(ii) Sketch the graph of $r = 1 - 2\cos\theta$ from $\theta = 0$ to $\theta = 2\pi$.