OBJECTIVES

This main objective of this experiment is to analyze, determine and compare the stresses systems for the thin and thick walled pressure vessels or cylinders.

LEARNING OUTCOMES (N.B Students **should not include** these as part of their report)

At the end of this laboratory session students should be able to

1. Apply the thin and thick cylinders theoretical concepts to determine the principal stresses and make comparison between the respective stress components
2. Determine the magnitude of principal stresses varies with internal pressure in thin cylinder.
3. Analyze the principal stresses distributions across the wall of thick cylinder when subject to internal pressure.
4. Write a clear and well presented laboratory report that describes the stress system in a thin and thick cylinder.

THEORETICAL BACKGROUND

Thin cylinder Theory

It may be shown that, for the thin cylinder pressure vessel, the hoop, longitudinal and radial stresses are given by (provided ratio D/t > 20), D = Internal diameter, t = wall thickness);

$$\sigma_H = \frac{PD}{2t} \quad , \quad \sigma_L = \frac{PD}{4t} = \frac{\sigma_H}{2} \quad \text{and} \quad \sigma_R = -P \approx 0$$

It is a normal convention to assume that magnitude of radial stress $\sigma_R <<< \sigma_L = \sigma_H/2$ and the hoop and longitudinal or axial stresses of the thin cylinder system are generally ***constant*** throughout the wall thickness.

Note: The given SM1007 cylinder system has a ratio of approximately 27, which is well above the ratio for being considered thin.

Theory of Thick Cylinder

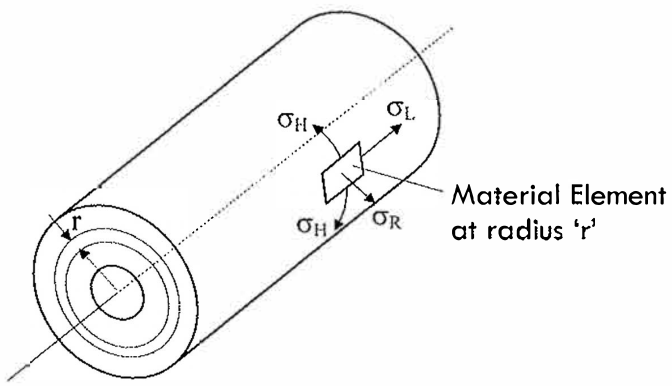

Figure 1 Thick cylinder under Internal Pressure, P (MPa)

Figure 1 shows a thick-walled cylinder subjected to an internal pressure P (MN/m^2). The figure details an element of material at some radius r, contained within an elemental cylinder. The assumption that the hoop and radial stresses are constants as in the case of thin cylinder theory is no longer valid for the thick-cylinder problems. Under ***opened-end condition***, the longitudinal stress σ_L may be ignored ($\sigma_L = 0$) and only a bi-axial stress system with be generated across the wall of the cylinder cross-section. It may be shown that, based on ***Lame's equations***, the Hoop and Radial stresses of the cylinder system are given by;

$$\sigma_H = A + \frac{B}{r^2} \quad .....(1) , \qquad \sigma_R = A - \frac{B}{r^2} \quad ......(2)$$

where constants A and B may be defined by using the ***boundary conditions*** of the cylinder.

Hence, by knowing constants A and B, the stresses variation throughout the wall of the cylinder may be obtained as shown below and in Figure 2.

Maximum σ_R occurs at the inner radius (R_1) i.e. $\sigma_R = -P$

Minimum σ_R occurs at the outer radius (R_2) i.e. $\sigma_R = 0$

Maximum σ_H occurs at the inner radius (R_1) i.e. $\sigma_H = P \cdot \frac{(K^2+1)}{(K^2-1)}$ (3)

Minimum σ_H occurs at the outer radius (R_2) i.e. $\sigma_H = \frac{(2P)}{(K^2-1)}$ (4)

where $K = \frac{R_2}{R_1}$

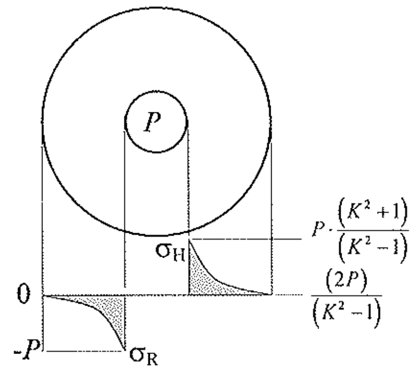

Figure 2: Distributions of σ_H and σ_R throughout cylinder wall due to P

Now for a cylinder under internal pressure P and free from axial loading ($\sigma_L = 0$), the maximum shear stress will occur at the inner radius, i.e r = R_1.

i.e. Maximum shear stress, $\tau_{max} = \frac{\sigma_R - \sigma_H}{2}$ (3)

Therefore, $\tau_{max} = \frac{P \cdot K^2}{(K^2-1)}$ (4)

where K = R_2/R_1. In the case of the given TQ cylinder: K = 4.054 and therefore τ_{max} =1.065P.

Note: For the thin and thick cylinders, all stresses are also referred as ***principal stresses.***

APPARATUS

Thin Cylinder

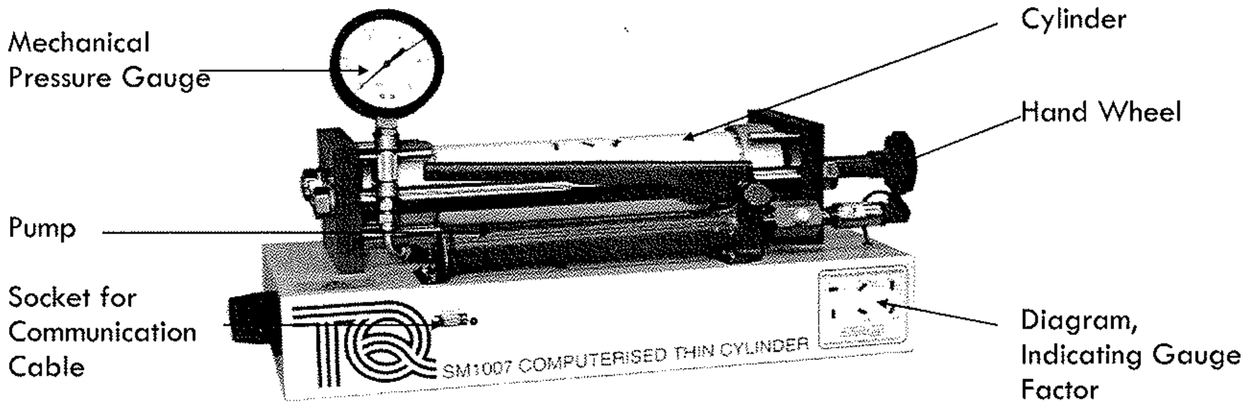

Figure 3 Layout of the SM1007

1. Figure 3 shows the SM1007 Thin Cylinder apparatus. It consists of a thin walled aluminum cylinder of 80 mm inside diameter and 3 mm wall thickness. Operating the hydraulic pump pressurizes the cylinder with oil.
2. The cylinder has six sensors on its surface that measure strain. A mechanical gauge and electronic sensor measure the hydraulic pressure in the cylinder. The cylinder is held in sturdy frame in which it is free to move along its axis. The strain (and thus the stress) can be measured with the cylinder in two configurations:
   a. "Open" ends – where the axial loads are taken by the frame (not the cylinder), therefore there is no direct axial stress
   b. "Closed" ends – where the axial loads are taken by the cylinder, therefore there must be direct axial stress

The two configurations are achieved using the large hand wheel at the end of the frame.

3. In the "open" ends condition the hand wheel is screwed fully in. This pushes the two pistons away from the cylinder end caps so that there is no contact between them. Therefore, the axial force is transmitted from the pressurized oil into the frame rather than the cylinder. See Figure 4.

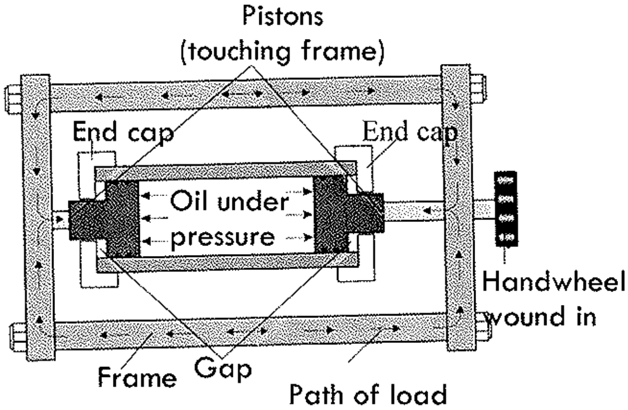

Figure 4 Open Ends Condition

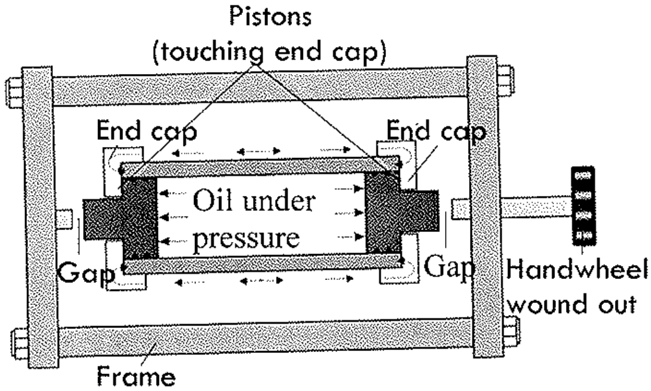

Figure 5 Closed Ends Condition

4. In the "closed" ends condition the hand wheel is wound out. This allows the pistons to move outward against the cylinder end caps so that there is no contact with the frame. Therefore the axial force is transmitted from the pressurized oil into the cylinder itself.

5. In relation to stress analysis, cylinders are divided into two groups: thin and thick. The distinction between the two relates to the ratio of internal diameter to wall thickness of a particular cylinder.

TASK 1:

Experiment A – Thin Cylinder with Open Ends

Conduct the experiment as demonstrated by the lab's instructor to get the necessary data

Experiment B - Thin Cylinder with Closed Ends

1. Calculate theoretical principal strains with a pressure 3 MPa, a Poisson's ratio, $\nu = 0.33$ and a Young's Modulu,s $E = 70$ MPa.
2. Validate these strains through your experimental work.
3. Conduct the experiment as demonstrated by the lab's instructor

TASK 2

Experiment C – Thick Cylinder

- Your experiment must be designed and conducted to investigate and determine the variation of hoop, radial and longitudinal stresses across the wall of thin and thick pressure vessels.

- All necessary data and results must be measured, recorded or tabulated systematically to facilitate its analysis and interpretation at the later stage of your work.

- Comparison of experimentally determined results with the theoretical results must also be presented, analyzed and discussed in your report.

Note: In all calculations for thick cylinder case, the following values for Young's Modulus and Poisson's ratio must be used; $E = 73.1$ GPa $\nu = 0.33$