**OBJECTIVES**

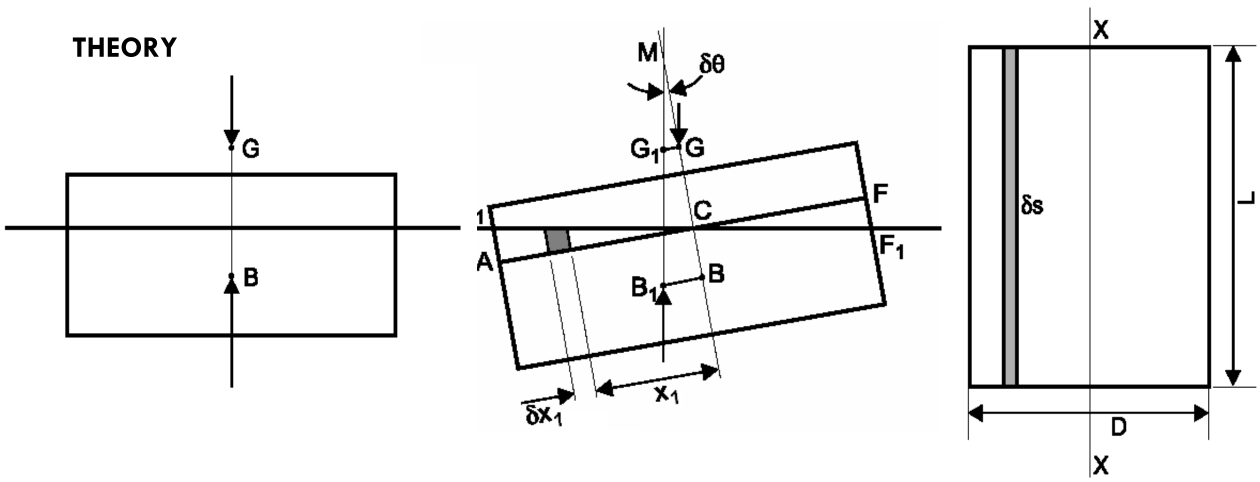
To determines the stability of a pontoon with its center of gravity at various heights.

**LEARNING OUTCOME**

At the end of this lab session, students should be able to

1. Understand the parameter affecting the stability of a floating body
2. Determine the stability of a floating body with different CG height

**THEORY**




(a) (b) (c)

**Figure 1** Derivation of Stability of Floating Pontoon

Consider the rectangular pontoon shown floating in equilibrium on an even keel, as shown in the cross section of Figure 1(a). The weight of the floating body acts vertically downwards through its centre of gravity $G$ and this is balanced by an equal and opposite buoyancy force acting upwards through the centre of buoyancy $B$, which lies at the centre of gravity of the liquid displaced by the pontoon.

To investigate the stability of the system, consider a small angular displacement $\delta\theta$ from the equilibrium position as shown on Figure 1(b). The centre of gravity of the liquid displace by the pontoon shifts from $B$ to $B_1$. The vertical line of action of the buoyancy force is shown on figure and intersects the extension of line $BG$ at $M$, the metacentre.

The equal and opposite forces through $G$ and $B_1$ exert a couple on the pontoon, and provided that $M$ lies above $G$ (as shown in Figure 1(b)) this couple acts in the sense of restoring the pontoon to even keel, i.e. the pontoon is stable. If, however, the metacentre $M$ lies below the centre of gravity $G$, the sense of the couple is to increase the angular displacement and the pontoon is unstable. The special case of neutral stability occurs when $M$ and $G$ coincides.

Figure 1(b) shows clearly how the metacentric height $GM$ may be established experimentally using the adjustable weight (of mass $\omega$) to displace the centre of gravity sideways from $G$. Suppose the adjustable weight is moved a distance $\delta x_1$ from its central position. If the weight of the whole floating assembly is $W$, then the corresponding movement of the centre of gravity of the whole in a direction parallel to the base of the pontoon is $\frac{\omega}{W}\delta x_1$. If this movement produces a new equilibrium position at an angle of a list $\delta\theta$, then in Figure 1(b), $G_1$ is the new position of the centre of gravity of the whole, i.e.

$$GG_1 = \frac{\omega}{W}\delta x_1 \tag{1}$$

Now, from the geometry of the figure:

$$GG_1 = GM\delta\theta \tag{2}$$

Eliminating $GG_1$ between these equations we derive:

$$GM = \frac{\omega}{W}\frac{\delta x_1}{\delta\theta} \tag{3}$$

or in the limit:

$$GM = \frac{\omega}{W}\frac{dx_1}{d\theta} \tag{4}$$

The metacentric height may thus be determined by measuring ($\frac{dx_1}{d\theta}$) knowing $\omega$ and $W$. Quite apart from experimental determinations, $BM$ maybe calculated from the measurement of the pontoon and the volume of liquid which it displaces. Referring again to Figure 1(b), it may be noted that the restoring moment about $B$, due to shift of the centre of buoyancy to $B_1$, is produced by additional buoyancy represented by triangle $AA_1C$ to one side of the centre line, and reduced buoyancy represented by triangle $FF_1C$ to the other.

The element shaded in Figure 1(b) and (c) has and area $\delta s$ in plan view and a height $x\delta\theta$ in vertical section, so that its volume is $x\delta s\delta\theta$. The weight of liquid displaced by this element is $wx\delta s\delta\theta$, where w is the specific weight of the liquid, and this is the additional buoyancy due to the element.

The moment of this elementary buoyancy force about $B$ is $wx^2\delta s\delta\theta$, so that the total restoring moment about $B$ is given by the expression:

$$w\delta\theta\int x^2 ds$$

where the integral extends over the whole area of the pontoon at the plane of the water surface. The integral may be referring to as $I$, where:

$$I = \int x^2 ds \tag{5}$$

where $I$ is the second moment of area of s about the axis X-X.

The total restoring moment about $B$ may also be written as the total buoyancy force, $wV$, in which $V$ is the volume of liquid displaced by the pontoon, multiplied by the lever arm $BB_1$. Equating this product to the expression for total restoring moment derived above:

$$wVBB_1 = w\delta\theta \int x^2 ds \qquad (6)$$

Substituting from Equation 5 for the integral and using the expression:

$$BB_1 = BM\delta\theta \qquad (7)$$

which follows from the geometry of Figure 1(b), leads to:

$$BM = \frac{I}{V} \qquad (8)$$

This result, which depends only on the measurement of the pontoon and the volume of liquid which it displaces, will be used to check the accuracy of the experiment.

It applies to a floating body of any shape, provided that $I$ is taken about an axis through the centroid of the area of the body at the plane of the water surface, the axis being perpendicular to the place in which angular displacement takes place. For a rectangular pontoon, $B$ lies at a depth below the water surface equal to half the total depth of immersion, and $I$ may readily be evaluated in terms of the dimensions of the pontoon as:

$$I = \int x^2 ds = \int_{-D/2}^{D/2} x^2 L dx = \frac{1}{12} LD^3 \qquad (9)$$

## APPARATUS

The arrangement of the apparatus is shown in Figure 2 below. A pontoon of rectangular form floats in water and carries a plastic sail, with five rows of V-slots at equivalent spaced heights on the sail. The slots' centers are spaced at 7.5 mm intervals, equally disposed about the sail centre line. An adjustable weight, consisting of two machined cylinders which can be screwed together, fits into the V-slots on the sail; this can be used to change the height of the center of gravity and the angle of list of the pontoon. A plumb bob is suspended from the top center of the sail and is used in conjunction with the scale fitted below the base of the sail to measure the angle of list.

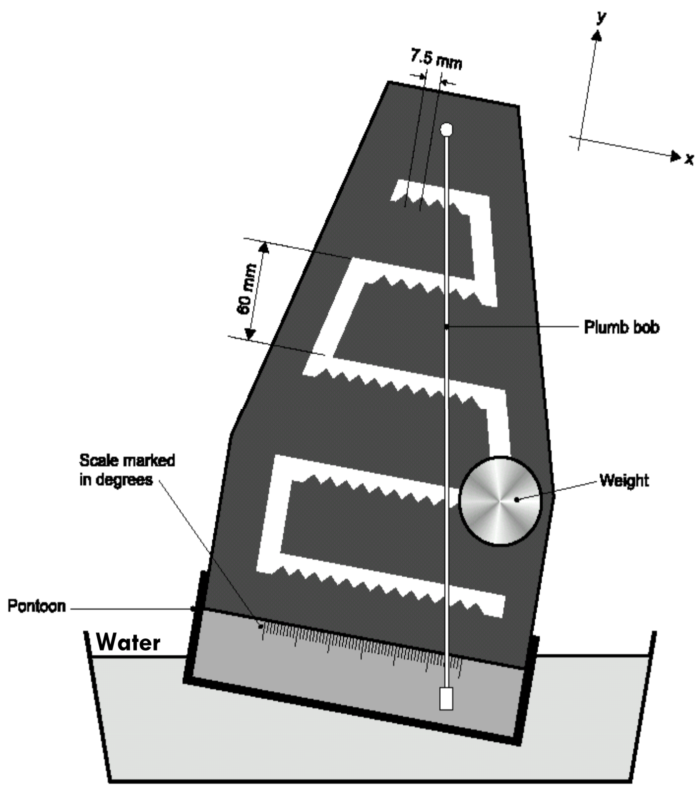



**Figure 2** Arrangement of Floating Pontoon

## PROCEDURES

A. To find the height of the centre of gravity $\bar{y}_i$ of a pontoon (refer to Figure 3).
   1. Fit the two magnetic weights to the base of the pontoon.
   2. Fit the thick knotted cord, with the plumb weight, through the hole in the sail, ensuring that the plumb weight is free to hang down on the side of the sail which has the scored centre line.
   3. Clamp the adjustable weight into the V-slot, on the centre line of the lowest row.
   4. Lift the pontoon by holding the plumb weight. Mark the point where the plumb line crosses the sail centre line with liquid-paper.
   5. Repeat step 3 for the other four rows.

B. To measure angle of list, $\theta$:
   1. For the highest row, displace the adjustable weight laterally from the sail centre line, with an increment of 15 mm at a time, until the plumb bob goes beyond the degree scale. Record the angle of list, $\theta$ for each increment into Table 1.

Note: Decide which side of the sail centre line is to be termed negative and then term list angles on that side negative.

2. Repeat step 1 for the other four rows to complete Table 1.

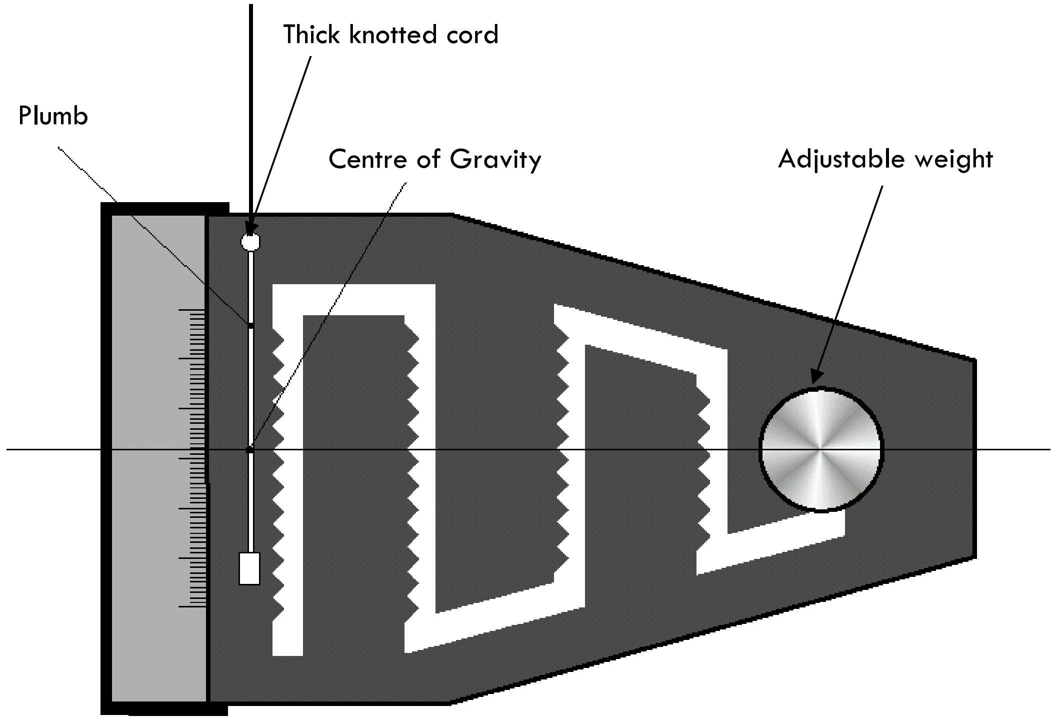



**Figure 3** Method of finding centre of gravity

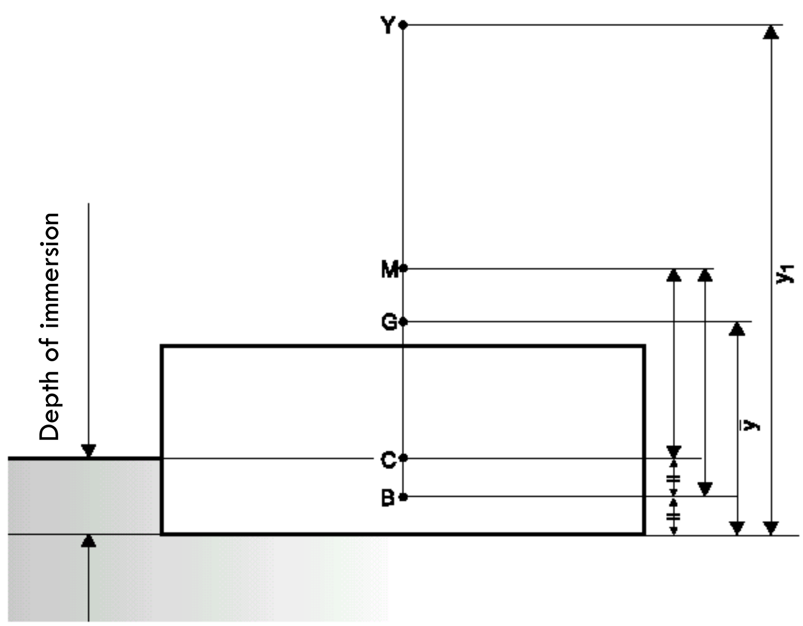



**Figure 4** Dimensions of Pontoon

Labels of dimensions of the pontoon are given in Figure 4, which are to be referred in the calculation section.

## STABILITY OF A FLOATING BODY

Name: ____________________________________ Metric Number: ___________________

Section / Group: ___________________________ Date of experiment: _______________

### EXPERIMENTAL DATA

The total weight of the apparatus (including the two magnetic weights) is stamped on a label affixed to the sail housing. The adjustable weight (☐) has its weight engraved on its side. The addition of these two values will give the total weight W of the pontoon.

| | | |
|---|---|---|
| Total weight of floating assembly (W) | =_______________ | kg |
| Adjustable weight ( $\omega$ ) | = _______________ | kg |
| Breadth of pontoon (D) | = _______________ | mm |
| Length of pontoon (L) | = _______________ | mm |
| Second moment of area I = $\frac{LD^3}{12}10^{-12}$ | = _______________ | $m^4$ |
| Volume of water displaced V = $\frac{W}{\rho}$ | = _______________ | $m^3$ |
| Height of metacentre above centre of buoyancy BM = $\frac{1}{V}$ | = _______________ | m |
| Depth of immersion of pontoon = $\frac{V}{LD}$ | = _______________ | m |
| Depth of centre of buoyancy CB = $\frac{V}{2LD}$ | = _______________ | m |

**Table 1** List angles for height and position of adjustable weight

| Height of adjustable weight $y_i$ mm (i) | Angles of list ( $^0$ ) for adjustable weight lateral displacement from sail centre line $x_1$ mm (ii) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | -75 | -60 | -45 | -30 | -15 | 0 | 15 | 30 | 45 | 60 | 75 |
| | | | | | | | | | | | |
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**Table 2** Derivation of metacentric height from experimental results

| Height of adjustable weight $yi$ (mm) | Height of G $\bar{y}_i$ (mm) | Height of G above water surface CG $= \bar{y}_i$ - immersion depth (mm) | $\frac{dx_1}{d\theta}$ ( mm/$^0$ ) | Metacentric height GM $= \frac{\omega}{W}\frac{dx_1}{d\theta}$ (mm) | Height of M above water surface CM = CG+GM (mm) |
|---|---|---|---|---|---|
| | | | | | |
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**QUESTION**

1. Plot the value of $\frac{dx_1}{d\theta}$, which is the slope of the graph of $x_1$ against $\theta$, against the height of centre of gravity above water line, CG.

   Short report: please provide sample of calculations in different sheet.

2. Base on the graph of $\frac{dx}{d\theta}$ versus CG, when is the pontoon become unstable? Justify.

3. When is the pontoon is naturally stable?

4. Conclusion